12-1 The Counting Principle

Main Ideas

- Solve problems involving independent events.
- Solve problems involving dependent events.

New Vocabulary

outcome
sample space
event
independent events
Fundamental Counting Principle
dependent events

GET READY for the Lesson

The number of possible license plates for a state is too great to count by listing all of the possibilities. It is much more efficient to count the number of possibilities by using the Fundamental Counting Principle.

Independent Events An **outcome** is the result of a single trial. For example, the trial of flipping a coin once has two outcomes: head or tail. The set of all possible outcomes is called the **sample space**. An **event** consists of one or more outcomes of a trial. The choices of letters and digits to be put on a license plate are called **independent events** because each letter or digit chosen does *not* affect the choices for the others.

Reading Math

There are both *infinite* and *finite* sample spaces. Finite sample spaces have a countable number of possible outcomes, such as rolling a die. Infinite sample spaces have an uncountable number of possible outcomes, such as the probability of a point on a line.

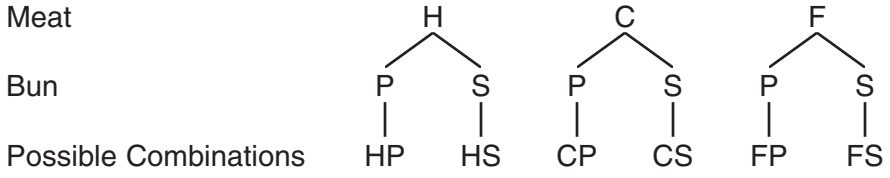
EXAMPLE 1 Independent Events

FOOD **A sandwich cart offers customers a choice of hamburger, chicken, or fish on either a plain or a sesame seed bun. How many different combinations of meat and a bun are possible?**

First, note that the choice of the type of meat does not affect the choice of the type of bun, so these events are independent.

Method 1 Tree Diagram

H represents hamburger, C, chicken, F, fish, P, plain, and S, sesame seed.



Method 2 Make a Table

Make a table in which each row represents a type of meat and each column represents a type of bun.

There are six possible outcomes.

		Bun	
		Plain	Sesame
Meal	Hamburger	HP	HS
	Chicken	CP	CS
	Fish	FP	FS

CHECK Your Progress

1. A cafeteria offers drink choices of water, coffee, juice, and milk and salad choices of pasta, fruit, and potato. How many different combinations of drink and salad are possible?

Notice that in Example 1, there are 3 ways to choose the type of meat, 2 ways to choose the type of bun, and 3 • 2 or 6 total ways to choose a combination of the two. This illustrates the **Fundamental Counting Principle**.

KEY CONCEPT — ***Fundamental Counting Principle***

Words If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.

Example If event M can occur in 2 ways and event N can occur in 3 ways, then M followed by N can occur in $2 \cdot 3$ or 6 ways.

This rule can be extended to any number of events.

STANDARDIZED TEST EXAMPLE — Fundamental Counting Principle

2 Kim won a contest on a radio station. The prize was a restaurant gift certificate and tickets to a sporting event. She can select one of three different restaurants and tickets to a football, baseball, basketball, or hockey game. How many different ways can she select a restaurant followed by a sporting event?

A 7 **B** 12 **C** 15 **D** 16

Test-Taking Tip

Remember that you can check your answer by making a tree diagram or a table showing the outcomes.

Read the Test Item

Her choice of a restaurant does not affect her choice of a sporting event, so these events are independent.

Solve the Test Item

There are 3 ways she can choose a restaurant and there are 4 ways she can choose the sporting event. By the Fundamental Counting Principle, there are 3 • 4 or 12 total ways she can choose her two prizes. The answer is B.

CHECK Your Progress

2. Dane is renting a tuxedo for prom. Once he has chosen his jacket, he must choose from three types of pants and six colors of vests. How many different ways can he select his attire for the prom?

F 9 **G** 10 **H** 18 **J** 36

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EXAMPLE — More than Two Independent Events

3 **COMMUNICATION** **Many answering machines allow owners to call home and get their messages by entering a 3-digit code. How many codes are possible?**

Reading Math

Independent and *dependent* have the same meaning in mathematics as they do in ordinary language.

The choice of any digit does not affect the other two digits, so the choices of the digits are independent events.

There are 10 possible first digits in the code, 10 possible second digits, and 10 possible third digits. So, there are 10 • 10 • 10 or 1000 possible different code numbers.

CHECK Your Progress

3. If a garage door opener has a 10-digit keypad and the code to open the door is a 4-digit code, how many codes are possible?

Dependent Events Some situations involve dependent events. With **dependent events**, the outcome of one event *does* affect the outcome of another event. The Fundamental Counting Principle applies to dependent events as well as independent events.

EXAMPLE Dependent Events

4 **SCHOOL** **Charlita wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?**

When Charlita schedules a given class for a given period, she cannot schedule that class for any other period. Therefore, the choices of which class to schedule each period are dependent events.

There are 6 classes Charlita can take during first period. That leaves 5 classes she can take second period. After she chooses which classes to take the first two periods, there are 4 remaining choices for third period, and so on.

Study Tip

Look Back

To review **factorials**, see Lesson 11-7.

Period	1st	2nd	3rd	4th	5th	6th
Number of Choices	6	5	4	3	2	1

There are $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 720 schedules that Charlita could have.

Note that $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6!$.

CHECK Your Progress

4. Each player in a board game uses one of six different pieces. If four players play the game, how many different ways could the players choose their game pieces?

CONCEPT SUMMARY

Independent Events	**Words**	If the outcome of an event does not affect the outcome of another event, the two events are independent.
	Example	Tossing a coin and rolling a die are independent events.
Dependent Events	**Words**	If the outcome of an event does affect the outcome of another event, the two events are dependent.
	Example	Taking a piece of candy from a jar and then taking a second piece without replacing the first are dependent events because taking the first piece affects what is available to be taken next.

CHECK Your Understanding

Examples 1–4 (pp. 684–686)

State whether the events are *independent* or *dependent*.

1. choosing the color and size of a pair of shoes
2. choosing the winner and runner-up at a dog show

Examples 1, 2 (pp. 684, 685)

3. An ice cream shop offers a choice of two types of cones and 15 flavors of ice cream. How many different 1-scoop ice cream cones can a customer order?

Example 2 (p. 685)

4. **STANDARDIZED TEST PRACTICE** A bookshelf holds 4 different biographies and 5 different mystery novels. How many ways can one book of each type be selected?

A 1 **B** 9 **C** 10 **D** 20

Example 3 (p. 685)

5. Lance's math quiz has eight true-false questions. How many different choices for giving answers to the eight questions are possible?
6. Pizza House offers three different crusts, four sizes, and eight toppings. How many different ways can a customer order a pizza?

Example 4 (p. 686)

7. For a college application, Macawi must select one of five topics on which to write a short essay. She must also select a different topic from the list for a longer essay. How many ways can she choose the topics for the two essays?

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–11	1, 4
12–26	1–4

State whether the events are *independent* or *dependent*.

8. choosing a president, vice-president, secretary, and treasurer for Student Council, assuming that a person can hold only one office
9. selecting a fiction book and a nonfiction book at the library
10. Each of six people guess the total number of points scored in a basketball game. Each person writes down his or her guess without telling what it is.
11. The letters A through Z are written on pieces of paper and placed in a jar. Four of them are selected one after the other without replacing any of them.
12. Tim wants to buy one of three different books he sees in a book store. Each is available in print and on CD. How many book and format choices does he have?
13. A video store has 8 new releases this week. Each is available on videotape and on DVD. How many ways can a customer choose a new release and a format to rent?
14. Carlos has homework in math, chemistry, and English. How many ways can he choose the order in which to do his homework?

Cross-Curricular Project

You can use the Fundamental Counting Principle to list possible outcomes in games. Visit algebra2.com to continue work on your project.

15. The menu for a banquet has a choice of 2 types of salad, 5 main courses, and 3 desserts. How many ways can a salad, a main course, and a dessert be selected to form a meal?
16. A baseball glove manufacturer makes gloves in 4 different sizes, 3 different types by position, 2 different materials, and 2 different levels of quality. How many different gloves are possible?
17. Each question on a five-question multiple-choice quiz has answer choices labeled A, B, C, and D. How many different ways can a student answer the five questions?

18. PASSWORDS Abby is registering at a Web site. She must select a password containing six numerals to be able to use the site. How many passwords are allowed if no digit may be used more than once?

ENTERTAINMENT For Exercises 19 and 20, refer to the comic strip. Assume that the books are all different.

Real-World Link

Before 1995, area codes had the following format.

(XYZ)

X = 2, 3, . . ., or 9

Y = 0 or 1

Z = 0, 1, 2, . . ., or 9

Source: www.nanpa.com

19. How many ways can you arrange the science books?

20. Since the science books are to be together, they can be treated like one book and arranged with the music books. Use your answer to Exercise 19 and the Counting Principle to find the answer to the problem in the comic.

AREA CODES For Exercises 21 and 22, refer to the information about telephone area codes at the left.

21. How many area codes were possible before 1995?

22. In 1995, the restriction on the middle digit was removed, allowing any digit in that position. How many total codes were possible after this change was made?

23. How many ways can six different books be arranged on a shelf if one of the books is a dictionary and it must be on an end?

24. In how many orders can eight actors be listed in the opening credits of a movie if the leading actor must be listed first or last?

25. HOME SECURITY How many different 5-digit codes are possible using the keypad shown at the right if the first digit cannot be 0 and no digit may be used more than once?

26. RESEARCH Use the Internet or other resource to find the configuration of letters and numbers on license plates in your state. Then find the number of possible plates.

EXTRA PRACTICE
See pages 916, 937.
MathOnline
Self-Check Quiz at algebra2.com

H.O.T. Problems

27. OPEN ENDED Describe a situation in which you can use the Fundamental Counting Principle to show that there are 18 total possibilities.

28. REASONING Explain how choosing to buy a car or a pickup truck and then selecting the color of the vehicle could be dependent events.

29. CHALLENGE The members of the Math Club need to elect a president and a vice president. They determine that there are a total of 272 ways that they can fill the positions with two different members. How many people are in the Math Club?

30. ***Writing in Math*** Use the information on page 684 to explain how you can count the maximum number of license plates a state can issue. Explain how to use the Fundamental Counting Principle to find the number of different license plates in a state such as Oklahoma, which has 3 letters followed by 3 numbers. Also explain how a state can increase the number of possible plates without increasing the length of the plate number.

STANDARDIZED TEST PRACTICE

31. ACT/SAT How many numbers between 100 and 999, inclusive, have 7 in the tens place?

A 90

B 100

C 110

D 120

32. REVIEW A coin is tossed four times. How many possible sequences of heads or tails are possible?

F 4

G 8

H 16

J 32

Spiral Review

33. Prove that $4 + 7 + 10 + \cdots + (3n + 1) = \frac{n(3n + 5)}{2}$ for all positive integers n. (Lesson 11-8)

Find the indicated term of each expansion. (Lesson 11-7)

34. third term of $(x + y)^8$

35. fifth term of $(2a - b)^7$

36. CARTOGRAPHY Edison is located at (9, 3) in the coordinate system on a road map. Kettering is located at (12, 5) on the same map. Each side of a square on the map represents 10 miles. To the nearest mile, what is the distance between Edison and Kettering? (Lesson 10-1)

Solve each equation by factoring. (Lesson 5-3)

37. $x^2 - 16 = 0$

38. $x^2 - 3x - 10 = 0$

39. $3x^2 + 8x - 3 = 0$

Solve each matrix equation. (Lesson 4-1)

40. $[x \quad y] = [y \quad 4]$

41. $\begin{bmatrix} 3y \\ 2x \end{bmatrix} = \begin{bmatrix} x + 8 \\ y - x \end{bmatrix}$

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.** (Lesson 11-7)

42. $\frac{5!}{2!}$

43. $\frac{6!}{4!}$

44. $\frac{7!}{3!}$

45. $\frac{6!}{1!}$

46. $\frac{4!}{2!2!}$

47. $\frac{6!}{2!4!}$

48. $\frac{8!}{3!5!}$

49. $\frac{5!}{5!0!}$